Algebra 2/Pre-Calculus

Name__________________

Using Normal PDF and CDF (Day 12, Statistics)

In this problem set, we will explore the Normal Probability Density Function (Normal PDF) and the Normal Cumulative Density Function (Normal CDF).

1. A bent coin lands heads 64% of the time and tails 36% of the time. We flip this coin 25 times.

a. What is the probability that the coin will land heads exactly 18 times?

b. You should have found that the probability that the coin landed heads 18 times was 0.1222 or 12.22%. In the rest of this problem, we will explore another way of answering this question. Start by finding the mean and standard deviation for the number of heads from the 25 flips.

c. On your calculator, press 2^{nd} and DISTR (DISTR is the 2^{nd} function of the VARS button). Choose 1: normalpdf(and enter the following: normalpdf(18, 16, 2.4). Press ENTER and write down the number that the calculator gives you.

d. You should have found that the answer you got in part **c** is close (but not exactly equal) to the answer you got in part **a**. This is because you have just solved the same problem by using the Normal Probability Density Function, which uses the bell curve to estimate probabilities. What was the significance of the 18, the 16, and the 2.4 that you plugged into the Normal PDF?

Answers a. 0.1222 b. mean = 16, standard deviation = 2.4 c. 0.1175 d. 18 was the number of heads we wanted, 16 was the mean, 2.4 was the standard deviation

2. In this question, we will continue to explore the Normal PDF.

a. Look back at problem **1**. The probability given by Normal PDF was 11.75% whereas the actual probability was 12.22%. These are pretty close, but clear not the same. Would increasing the number of flips increase the accuracy of the Normal PDF approximation? Explain why or why not. ***Hint:*** Remember, the Normal PDF uses the bell curve to estimate probabilities.

b. Suppose we use the same bent coin from the last problem (landing heads 64% of the time), but this time we flip the coin 100 times.

Without using Normal PDF, find the probability that the coin lands heads exactly 67 times.

c. Now use the Normal PDF to find the probability that the coin lands heads exactly 67 times. ***Hint:*** What information do you need to find before using Normal PDF?

d. Did increasing the number of flips make the Normal PDF method more accurate? Why do you think this happened? Explain.

Some answers a. The accuracy would increase as the number of flips increases. (As the number of flips increases, the probability histogram looks more and more like the bell curve.) b. $({}_{100}C_{67})(0.64)^{67}(0.36)^{33} = 6.94\%$ c. $\text{normalpdf}(67, 64, 4.8) = 6.84\%$ d. The accuracy increased for the reasons given in part **a**.

3. Now suppose we flip the bent coin 2500 times. (The coin still lands heads 64% of the time.)

a. Find the probability that the coin lands heads exactly 1600 times **without** using Normal PDF.

b. You probably found that the calculator gave you an overflow error. Our calculators simply can't handle numbers that are this big. But we can still do this problem using Normal PDF. Find the answer.

c. You should have found that the probability was 0.0166 or 1.66%. How accurate do you think this answer is? ***Hint:*** There are a lot of flips…

Answers a. If you try $({}_{2500}C_{1600})(0.64)^{1600}(0.36)^{900}$ on the calculator, you will probably get an error message. b. $\text{normalpdf}(1600,1600,24) = 1.66\%$ c. Very accurate. Here are both calculations listed to more decimal places: $({}_{2500}C_{1600})(0.64)^{1600}(0.36)^{900} = 0.0166207$ and $\text{normalpdf}(1600,1600,24) = 0.0166226$ (Note: Even though the calculator can't do the calculation in part a, wolframalpha can. That's how I got two answers for comparison in part c.)

4. Use Normal PDF to answer each of the following questions.

a. Suppose a coin lands heads 30% of the time. If the coin is flipped 300 times, what is the probability that the coin lands heads exactly 94 times?

b. Suppose a standard six sided die is rolled 100 times. What is the probability that a 1 is rolled exactly 15 times?

c. Suppose a multiple choice test has 120 questions. Each question has five possible answers (A, B, C, D, E). You have no clue how to do the problems, so you guess randomly on every question. What is the probability that you get exactly 25 questions right?

Answers a. $\text{normalpdf}(94, 90, 7.9373) = 4.43\%$ b. $\text{normalpdf}(15, 16.6667, 3.7268) = 9.69\%$

c. $\text{normalpdf}(25, 24, 4.3818) = 8.87\%$

Normal CDF

In the last group of problems, we explored how to use the Normal Probability Distribution Function (normal PDF) to estimate the probability of a specific event (such as getting exactly 67 heads when flipping a coin 100 times). We will now use the Normal Cumulative Distribution Function (normal CDF) to estimate the probability for a range of events (such as getting between 50 and 55 heads when flipping a coin 100 times).

5. A bent coin lands heads 64% of the time. We flip the coin 100 times.

a. What is the probability that the coin lands heads between 62 and 66 times? ***Suggestion:*** Use wolframalpha to find these probabilities quickly.

b. Use the normal curve to estimate the probability that the coin lands heads between 62 and 66 times. ***Hint:*** Start by finding the mean and the standard deviation. Then find the upper and lower bounds that you will need to enter into the calculator.

c. Here's another way of approaching this same problem. On your calculator, press 2nd and DISTR (DISTR is the 2nd function of the VARS button). Choose 2: normalcdf(and enter the following: normalcdf(62, 66, 64, 4.8). Press ENTER and write down the number that the calculator gives you.

d. How did your answer in parts **a** and **b** compare? What about your answers in parts **b** and **c**? Explain.

Answers a. Expand $(.64h + .36t)^{100}$ using wolframalpha and add up the relevant coefficients: $.0752 + .0806 + .0829 + .0816 + .0769 = .3972$ b. mean = 64, standard deviation = 4.8, lower limit = $\frac{62-64}{4.8} = -0.4167$, upper limit = $\frac{66-64}{4.8} = 0.4167$, answer = 0.3231 c. 0.3231 d. The answers in part **a** is the most accurate answer, whereas the answers for **b** and **c** are both estimates based on the normal curve.

In the last problem, we learned a new calculator procedure called normal CDF. ***Note:*** Normal CDF is an alternative to actually graphing the bell curve. It's a little easier to do, but we don't get to see the bell curve getting "shaded in."

6. Suppose you flip a fair coin 900 times.

a. What is the probability that the coin lands heads at least 465 times? ***Note:*** Use Normal CDF to find the answer.

b. You should have found that normalcdf(465, 900, 450, 15) = 0.1587. This means that there is a 15.87% chance that the coin will land heads at least 465 times. Now find the probability that the coin will land heads at least heads at least 475 times.

c. You should have found that normalcdf(475, 900, 450, 15) = 0.0478. This means that there is a 4.78% chance that the coin will land heads at least 475 times.

Now let's ask this question the opposite way: If there is a 10% chance that the coin will land heads at least x times, what is x? ***Note:*** In the next question, we will learn a technique for solving this type of question. But try it on your own first! ***Some hints:*** Could you solve this via guess and check? Could you solve this graphically?

7. In answering the last problem, you might have tried plugging a bunch of different values for x into the function normalcdf(x, 900, 450, 15). If you did this, you found that $x = 469$ yielded a probability slightly above 10% whereas $x = 470$ resulted in a probability just below 10%. Hence 469 and 470 are both reasonable answers.

It would be better if we didn't have to use guess and check to answer the question. In this problem, we develop a graphical method for tackling questions of this type.

a. Graph the following equation on your calculator: $y_2 = \text{normalcdf}(x, 900, 450, 15)$. Use the following window: $0 \le x \le 900$ and $0 \le y \le 1$. Sketch your graph below. ***Note:*** You should still have the equation of the normal curve entered in y_1, but turn it off while you are working on this problem.

b. Explain why the window $0 \le x \le 900$ and $0 \le y \le 1$ makes sense for this graph. ***Hint:*** What do the x values represent? What do the y values represent?

c. Why does the graph seem to "crash downwards" in the middle? Explain. ***Hint:*** Where is most of the area located on the bell curve? Near the mean? Far from the mean?

d. What value should we graph in y_3 to answer the question from problem **6c**?

e. Graph $y_3 = 0.1$ and determine where this line intersects the graph of $y_2 = \text{normalcdf}(x, 900, 450, 15)$. How is this intersection related to the question in problem **6c**?

Answer e. $x = 469.22$