Algebra 2/Pre-Calculus

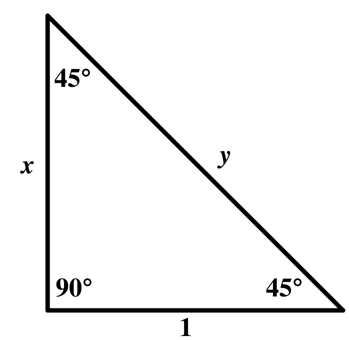
Name__________________

Special Right Triangles (Day 2, Right Triangle Trigonometry)

In this problem set, we will explore the ratio of the sides for special right triangles, specifically the 30-60-90 triangle and the 45-45-90 triangle. We will also learn about how inverse trigonometric functions can be used to find angles. Finally, we will introduce three more trigonometric functions: secant, cosecant, and cotangent.

1. The goal of this problem is to find the ratio of the sides for a 45-45-90 triangle **without using trigonometry**. (You can use ideas from geometry, but don't use sine or cosine or tangent.)

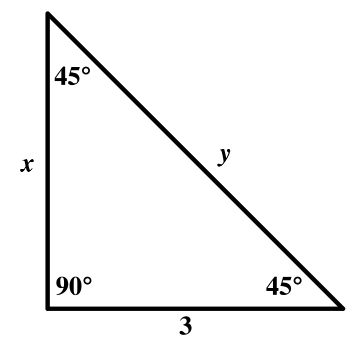
a. Find the value of x. Explain how you got your answer.



b. Find the value of y. Explain how you got your answer.

Answers a. The triangle is isosceles, so $x = 1$. b. From the Pythagorean theorem, $1^2 + 1^2 = y^2$, so $y = \sqrt{2}$

2. Here's another 45-45-90 right triangle. Find the values of x and y. Explain how you got your answer.



Answer This triangle has dimensions that are 3 times as big as the triangle in the last problem, so $x = 3$ and $y = 3\sqrt{2}$.

3. Consider the following statements.

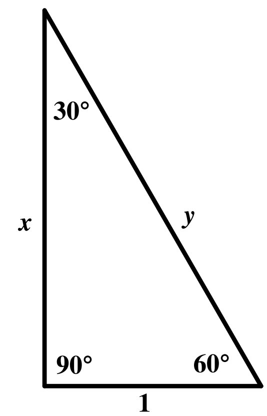
Statement 1: "The sides of a 45-45-90 triangle are always 1, 1, and $\sqrt{2}$."

Statement 2: "The sides of a 45-45-90 triangle are always in a ratio of 1, 1, and $\sqrt{2}$."

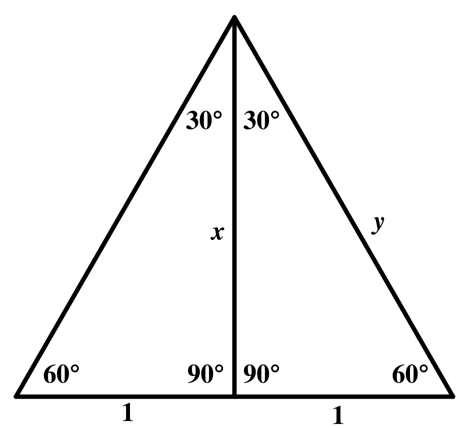
Which statement is correct? Why?

4. The goal of this problem is to find the ratio of the sides for a 30-60-90 triangle **without using trigonometry and without using your calculator**. (You can use ideas from geometry, but don't use sine or cosine or tangent.)

a. Find the values of x and y. Explain how you got your answer. ***Note:*** This one is tricky! If you want a hint, look ahead to part **b**. But try it on your own first.

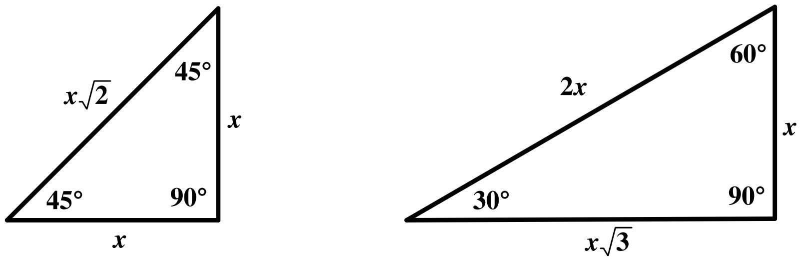


b. The trick to solving the 30-60-90 triangle is to draw a second 30-60-90 triangle next to it, as shown below. Explain how you know that the big triangle is equilateral. Then try find the values of x and y.



Answers All of the angles in the big triangle are 60°, so it is equilateral. Each side of the big triangle has a length of 2, so $y = 2$. Finally, the Pythagorean theorem tells us that $1^2 + x^2 = 2^2$, so $x = \sqrt{3}$.

We have found that the sides of a 45-45-90 triangle are in a ratio of $1{:}1{:}\sqrt{2}$ and the sides of a 30-60-90 triangle are in a ratio of $1{:}\sqrt{3}{:}2$, as summarized in the diagram below. Make sure to memorize these ratios, as we will be using them throughout the rest of this unit.



5. Find each of the following **without using your calculator**. ***Hint:*** Start by drawing a right triangle.

a. $\sin 60^\circ$

b. $\sin 30^\circ$

c. $\cos 60^\circ$

d. $\tan 60^\circ$

Answers a. $\frac{\sqrt{3}}{2}$ b. $\frac{1}{2}$ c. $\frac{1}{2}$ d. $\sqrt{3}$

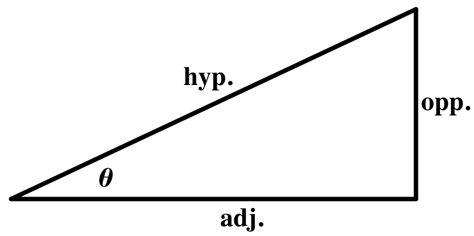
6. Olivia and Lewis were trying to find the value of $\sin 45^\circ$. Olivia said the answer was $\frac{1}{\sqrt{2}}$ and Lewis said the answer was $\frac{\sqrt{2}}{2}$. Who was right?

Answer They are both right: $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

7. What is the value of $\tan 30^\circ$? Write your answer two different ways.

Answer $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Along with sine, cosine, and tangent, there are three more trig functions that we will use sometimes. They are called secant, cosecant, and cotangent. We define them below.



Definitions In a right triangle, we define secant, cosecant, and cotangent in the following way:

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

8. Find each of the following. ***Hint:*** Start by drawing a right triangle.

a. $\sec 45^\circ$

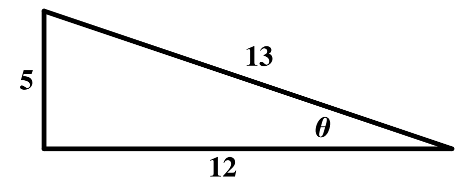
b. $\csc 30^\circ$

c. $\cot 60^\circ$

d. $\cot 45^\circ$

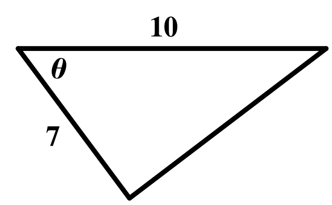
Answers a. $\sqrt{2}$ b. 2 c. $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ d. 1

9. Find the values of $\cos\theta$, $\sin\theta$, $\tan\theta$, $\sec\theta$, $\csc\theta$, and $\cot\theta$ in the right triangle pictured below.



Answers $\cos\theta = \frac{12}{13}$, $\sin\theta = \frac{5}{13}$, $\tan\theta = \frac{5}{12}$, $\sec\theta = \frac{13}{12}$, $\csc\theta = \frac{13}{5}$, $\cot\theta = \frac{12}{5}$

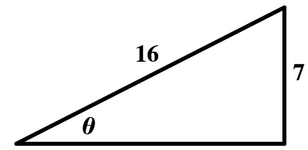
10. Find the values of $\cos\theta$, $\sin\theta$, $\tan\theta$, $\sec\theta$, $\csc\theta$, and $\cot\theta$ in the right triangle pictured below.



Answers $\cos\theta = \frac{7}{10}$, $\sin\theta = \frac{\sqrt{51}}{10}$, $\tan\theta = \frac{\sqrt{51}}{7}$, $\sec\theta = \frac{10}{7}$, $\csc\theta = \frac{10}{\sqrt{51}} = \frac{10\sqrt{51}}{51}$, $\cot\theta = \frac{7}{\sqrt{51}} = \frac{7\sqrt{51}}{51}$

11. Consider the following right triangle (drawn to scale below).

a. Without using your calculator, estimate the value of θ.



b. Find the value of θ by using the inverse trigonometric functions on your calculator. ***Note:*** If you don't remember how to do this, you can look ahead to part **c**.

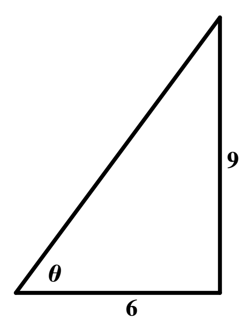
c. Here's a solution to the last problem:

$$\sin\theta = \tfrac{7}{16}$$

$$\theta = \sin^{-1}(\tfrac{7}{16})$$

$$\theta = 25.94^\circ$$

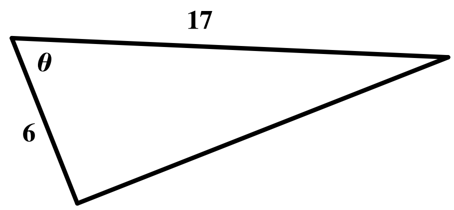
Use a similar method to find the value of θ in the right triangle pictured below. ***Note:*** Make sure your calculator is in degree mode.



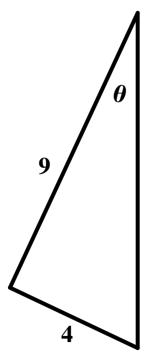
Answer c. $\theta = 56.31^\circ$

12. Find the value(s) for the variables in each of the following right triangles.

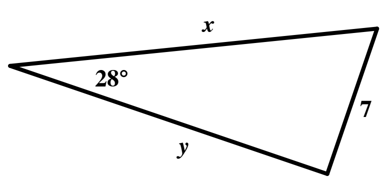
a.



b.



c.



Answers a. 69.33° b. 23.96° c. $x = 14.91$ and $y = 13.17$