Algebra 2/Pre-Calculus **Name__________________**

Repeated Experiments (Day 5, Statistics)

In this handout, you will investigate means, variance, and standard deviation for repeated experiments.

1. Suppose you have a standard six sided die. (The faces are labeled {1, 2, 3, 4, 5, 6} and each number is equally likely.)

a. Find the mean of value for this die.

b. Now consider an "unfair" die. The faces are still labeled 1 through 6, but the die is weighted so that there is a 50% chance of getting a 5 and a 10% chance for each of the remaining numbers. Find the mean for the unfair die. ***Hint:*** You find the mean in the same way you would find the expected value.

c. Hopefully, you found the mean for the unfair die by performing the following calculation: $1 \cdot 0.1 + 2 \cdot 0.1 + 3 \cdot 0.1 + 4 \cdot 0.1 + 5 \cdot 0.5 + 6 \cdot 0.1 = 4.1$. Notice that what we've really done here is an expected value calculation. The word "mean" is a synonym for "average value" which is in turn a synonym for "expected value." Calculating a mean is identical to calculating expected value.

Another six sided die has faces labeled 1 through 6, but this time, the probability of getting a 1 is $p(1)$, the probability of getting a 2 is $p(2)$ and so on. Find the mean for this die. (Your answer will involve the values $p(1), p(2), ..., p(6)$.)

In the last problem, you should have found that the mean for this die was given by the calculation: $1 \cdot p(1) + 2 \cdot p(2) + 3 \cdot p(3) + 4 \cdot p(4) + 5 \cdot p(5) + 6 \cdot p(6)$. This suggests a formula we could always use when finding a mean. We give this formula below as a definition.

Definition Mean: Suppose an event has possible (numerical) outcomes are $x_1, x_2, \ldots, x_n$ with corresponding probabilities $p(x_1), p(x_2), \ldots, p(x_n)$.

Then $\bar{x} = x_1 \cdot p(x_1) + x_2 \cdot p(x_2) + x_3 \cdot p(x_3) + \ldots + x_n \cdot p(x_n)$.

This formula can also be written using sigma notation: $\bar{x} = \sum_{i=1}^{n} x_i \cdot p(x_i)$.

Note that this formula is a *generalization* of our previous formula for finding the mean. When outcomes were *equally likely*, we used the formula $\bar{x} = \frac{1}{n}(x_1 + x_2 + x_3 + \ldots + x_n)$. Our new definition allows us to find the outcomes aren't equally likely (as in the unfair die, for example).

2. When events are equally likely, we should get the same value for the mean whichever way we calculate it. Calculate the mean value for a standard six sided die using our new formula.

3. Find the variance of the unfair die from problem **1b**. ***Remember:*** The variance is the mean of the squared deviations, so you will need to do a calculation using the new formula.

4. You should have found that the variance for the unfair die was 2.29. (You can find this by doing the calculation:

$$(1-4.1)^2 \cdot 0.1 + (2-4.1)^2 \cdot 0.1 + (3-4.1)^2 \cdot 0.1 + (4-4.1)^2 \cdot 0.1 + (5-4.1)^2 \cdot 0.5 + (6-4.1)^2 \cdot 0.1 = 2.29$$

Find the standard deviation for the unfair die.

5. Suppose we roll the unfair die three times. What is the mean value for the sum? What is the variance for the sum? What is the standard deviation for the sum? ***Hint:*** We have already learned a shortcut for finding these values.

6. Suppose we are flipping a fair coin. What are the mean, variance, and standard deviation for the number of heads we would get from a single flip? ***Hint:*** We have a 50% chance of getting 1 head and a 50% chance of getting 0 heads.

7. Suppose we flip the fair coin 15 times. What are the mean, variance, and standard deviation for the number of heads?

8. Suppose our coin is slightly bent so that there is a 55% chance of getting heads and a 45% chance of getting tails.

a. What are the mean, variance, and standard deviation for the number of heads from a single flip?

b. What are the mean, variance, and standard deviation for the number of heads if we flip this coin 100 times?