Algebra 2/Pre-Calculus

Name_____

Formulas for Bernoulli Trials (Day 8, Statistics)

Let's start by reviewing some of the key ideas from the last handout:

- Repeated binomial experiments (like coin flipping) follow a specific shape, called the normal distribution (the "bell" curve). The more flips we do, the "smoother" this distribution gets.
- The mean is at the center of the bell curve. This means that values close to the mean are much more likely than values that are far from the mean.
- These results are true both when events are equally likely (the fair coin) and when events are not equally likely (the unfair coin).

How do we measure how likely or unlikely it might be for us to get a certain outcome? One way we could do this would be to measure how many standard deviations we are from the mean. (If our event is many standard deviations from the mean, it is very unlikely). Our goal in this handout is to learn how to do this.

1. Suppose we have an unfair coin that lands heads 20% of the time and lands tails 80% of the time.

a. Find the mean, variance, and standard deviation for the number of heads we would get from a single flip of this coin.

b. First, check your answers for the last problem: mean = $(1)(0.2)+(0)(0.8)=0.2$, variance = $(1-0.2)^2(0.2)+(0-0.2)^2(0.8)=0.16$, standard deviation = $\sqrt{0.16}=0.4$

Now find the mean, variance, and standard deviation for the number of heads we would get from 5 flips of this coin.

c. Find the mean, variance, and standard deviation for the number of heads we would get from n flips of this coin. ***Note:*** Your answer will involve n.

2. Now suppose we have an unfair coin with probability p of landing heads. (In the last problem, p was 0.20.)

a. What is the probability that the coin will land tails? (You may assume that the coin always lands heads or tails. It never lands on its side or flies off into outer space, etc.)

b. You should have found that coin hand a $1 - p$ probability of landing tails. (To see this, think about the example from problem **1**. There was a 0.2 probability of landing heads, so there was a $1 - 0.2 = 0.8$ probability of landing tails.) Now find the mean number of heads from a single flip. ***Note:*** Just as in part **a**, your answer will involve p.

c. Find the mean number of heads for n flips of this coin.

d. Find the variance for the number of heads from a single flip of this coin. ***Note:*** This is a bit harder. Think back to the way you calculated variance in problem **1a**. Don't worry about simplifying your answer – yet.

e. You should have found that the variance (for a single flip) was given by the following formula: $(1 - p)^2 p + (0 - p)^2 (1 - p)$. (If you didn't get something equivalent to this, check your work for part **e**.) Now simplify this expression.

f. You should have found that the expression in part **e** simplified to $p(1 - p)$. (If you didn't get this, check your work for the last problem.)

g. Find the variance for the number of heads from n flips of this coin.

h. Find the standard deviation for the number of heads from n flips of this coin.

The results of the last problem are very important. Let's summarize them before we go on. We will also use this as an opportunity to introduce some vocabulary.

Definition Bernoulli trial: A Bernoulli trial is an experiment with two outcomes, success and failure. (Think of flipping the coin – success is getting heads, failure is getting tails.) A Bernoulli trial has a probability of success p, so the probability of failure is $1 - p$.

Note: This definition is given on page 247 of the textbook.

Suppose we have a Bernoulli trial with a probability p of success. In n trials, we have the following results (which you found in the last problem):

mean = np

variance = $np(1 - p)$

standard deviation = $\sqrt{np(1 - p)}$

Note: These formulas are covered on page 249 of the textbook.

Take a moment to make sure these formulas match the formulas you got. We will be using these formulas a lot throughout the rest of this unit.

3. Use the formulas listed above to find each of the following:

a. A coin is bent so that it has a 60% chance of landing heads. Find the mean, variance, and standard deviation for the number of heads you would get flipping the coin 30 times.

b. You roll a fair, standard six-sided die 50 times and count the number of times you get a 3. What are the mean, variance, and standard deviation for the number of 3's you get?

c. You are taking a multiple choice test. There are 100 questions and each question has five possible answers (A, B, C, D, or E). You have no clue what the answers are, so you guess randomly. Find the mean, variance, and standard deviation for the number of questions you get right. ***Hint:*** For each question, what is the probability that you get it right?

Answers a. Mean = 18, Variance = 7.2, Standard deviation = 2.683 b. Mean = 8.333, Variance = 6.944, Standard deviation = 2.635 c. Mean = 20, Variance = 16, Standard deviation = 4

4. Suppose we have a bent coin that lands heads 55% of the time. We flip the coin ten times and count the number of heads we get.

a. What is the probability that we get exactly 7 heads? ***Hint:*** Look back to problems **5f** and **5g** on "Day 6 handout" if you get stuck.

b. You should have found that the answer to the last problem was $\binom{10}{7}(0.55)^7(0.45)^3 = .1665$. Now find the probability of getting *at least* 7 heads.

c. You should have found that the answer to the last problem was $\binom{10}{7}(0.55)^7(0.45)^3 + \binom{10}{8}(0.55)^8(0.45)^2 + \binom{10}{9}(0.55)^9(0.45) + \binom{10}{10}(0.55)^{10} = 0.2660$.

Now find the mean, variance, and standard deviation for the number of heads you would get from flipping the coin 10 times. (Use the formulas from the previous page).

d. You should have found that the mean was 5.5 and the standard deviation was 1.573. How many standard deviations above the mean is the event of getting 7 heads?

e. You should have found that getting the event of getting 7 heads was 0.954 standard deviations above the mean because $\frac{7-5.5}{1.573} = 0.953$. How many standard deviations below the mean is the event of getting 4 heads? 2 heads? No heads?

5. Suppose we have an unfair coin that lands heads 62% of the time and lands tails 38% of the time. We flip the coin 40 times. ***Note:*** Answers are given at the end of this problem.

a. Find the mean, variance, and standard deviation for the number of heads from 40 flips.

b. What number of heads is one standard deviation above the mean? What number of heads is one standard deviation below the mean? ***Note:*** Your answer will involve decimals.

c. Which possible outcomes (numbers of heads) are within one standard deviation of the mean?

d. What number of heads is two standard deviation above the mean? What number of heads is two standard deviation below the mean? ***Note:*** Your answer will involve decimals.

e. Which possible outcomes (numbers of heads) are within two standard deviation of the mean?

Answers a. Mean = 24.8, Variance = 9.424, Standard Deviation = 3.070 b. One standard deviation above = $24.8 + 3.07 = 27.87$, One standard deviation below = $24.8 - 3.07 = 21.73$ c. 22, 23, 24, 25, 26, 27 d. Two standard deviations above = $24.8 + 2(3.07) = 30.94$, Two standard deviations below = $24.8 - 2(3.07) = 18.66$ e. 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30

6. Joseph was taking a multiple choice test but he hadn't prepared at all, so he had to guess randomly on every question. There were 50 questions on the test and each question had five possible answers (A, B, C, D, E).

a. Find the mean, variance, and standard deviation for the number of questions that Joseph answers correctly.

b. Which possible outcomes (numbers of correct answers) are within one standard deviation of the mean?

c. Which possible outcomes (numbers of correct answers) are within two standard deviation of the mean?

d. Which possible outcomes (numbers of correct answers) are more than three standard deviations above the mean?

Answers a. Mean = 10, Variance = 8, Standard Deviation = 2.828 b. 8, 9, 10, 11, 12

c. 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 d. Any number of correct answers 19 or above is more than three standard deviations above the mean.