Algebra 2/Pre-Calculus

Name__________________

Extra Problems (Day 8, Pascal's Triangle)

More Practice Problems!

1. Make a Pascal's Triangle in the space below. You triangle should include rows 0 through 10. Work neatly and carefully!

2. Suppose a coin is flipped 8 times. What is the probability that the coin will land heads exactly 5 times?

Answer $\frac{{}_{8}C_{5}}{2^{8}} = \frac{56}{256} = 0.219$

3. Suppose a coin is flipped 20 times. What is the probability that the coin will land heads exactly 7 times?

Answer $\frac{{}_{20}C_{7}}{2^{20}} = \frac{77520}{1,048,576} = 0.0739$

4. Suppose a coin is flipped 16 times. What is the probability that the coin will land heads at least 14 times?

Answer $\frac{{}_{16}C_{14}}{2^{16}} + \frac{{}_{16}C_{15}}{2^{16}} + \frac{{}_{16}C_{16}}{2^{16}} = \frac{137}{65,536} = 0.00209$

5. Suppose a coin is flipped 16 times. What is the probability that it will land hands less than 14 times? ***Hint:*** How is this question related to the last problem?

Answer $1 - \frac{137}{65,536} = \frac{65,399}{65,536} = 0.99791$

6. Suppose a coin is flipped 15 times. What is the probability that the coin will land tails less than 4 times?

Answer $\frac{{}_{15}C_0}{2^{15}} + \frac{{}_{15}C_1}{2^{15}} + \frac{{}_{15}C_2}{2^{15}} + \frac{{}_{15}C_3}{2^{15}} = \frac{576}{32,768} = 0.0176$

7. Mr. Verner has 20 students in his math class. He decides to select one student to present problem 1 at the whiteboard, another student to present problem 2, and a third student to present problem 3. How many ways can he select the three students?

Answer ${}_{20}P_3 = 20 \cdot 19 \cdot 18 = 6,840$

8. Explain why the answer to the last problem could be found from the calculation $20 \cdot 19 \cdot 18$.

Answer There are 20 possible students that Mr. Verner could select for the first problem, 19 students that Mr. Verner could select for the second problem, and 18 students that Mr. Verner could select for the last problem.

9. Mr. Verner has 20 students in his math class. This time, he decides to select a group of three students to present the problems together. How many ways can he select this group?

Answer $_{20}C_3 = \frac{20 \cdot 19 \cdot 18}{3 \cdot 2 \cdot 1} = 1{,}140$

10. Explain why the answer to the last problem could be found from the calculation $\frac{20 \cdot 19 \cdot 18}{3 \cdot 2 \cdot 1}$.

Answer Mr. Verner still has 20 choices for the first student, 19 choices for the second student, and 18 choices for the third student. But this time, the order doesn't matter. Any group of three students may be arranged in $3 \cdot 2 \cdot 1 = 6$ ways. Since the order doesn't matter, we must divide by 6. Thus, our answer is $\frac{20 \cdot 19 \cdot 18}{3 \cdot 2 \cdot 1} = 1{,}140$.

11. Is ${}_9P_3$ equal to $\frac{9!}{6!}$? Explain.

Answer They are equal. Here's why: $\frac{9!}{6!} = \frac{9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{6\cdot5\cdot4\cdot3\cdot2\cdot1} = 9\cdot8\cdot7 = {}_9P_3$

12. Which of the following two statements is correct? ${}_nP_r = \frac{n!}{(n-r)!}$ or ${}_nP_r = \frac{n!}{r!}$?

Answer ${}_nP_r = \frac{n!}{(n-r)!}$

13. A class has 9 students and a teacher. Three people from the class are selected to receive a prize. (The teacher may or may not be one of the people who is selected.)

Use this problem situation to explain why ${}_{10}C_3 = {}_9C_2 + {}_9C_3$.

Answer We know that the teacher may or may not be selected for the prize. There are ${}_9C_2$ ways to choose the winners if the teacher is selected and ${}_9C_3$ ways to choose the winners if the teacher is not selected. Hence, the total number of ways to select three people for the prize is ${}_9C_2 + {}_9C_3$. But since there are 10 people, we could also find the total number of ways to select three people for the prize by simply finding ${}_{10}C_3$. So it must be the case that ${}_9C_2 + {}_9C_3 = {}_{10}C_3$.

14. Where in Pascal's Triangle do we see the relation ${}_{10}C_3 = {}_9C_2 + {}_9C_3$? Explain.

Answer ${}_9C_2 = 36$ and ${}_9C_3 = 84$ are both entries in the 9^{th} row of Pascal's Triangle. ${}_{10}C_3 = 120$ is the entry immediately below on the 10^{th} row. ***Note:*** This is why Pascal's Triangle generates the combination numbers!

15. Jennifer and Jinyung were making a pizza together. Jennifer said, "Okay, I have peppers, onions, mushrooms, and pepperoni. We could make a pizza with all of these toppings. Or some of these toppings. Or none of these toppings." Jinyung started to wonder how many different pizzas were possible.

Use this problem situation to explain why ${}_4C_0 + {}_4C_1 + {}_4C_2 + {}_4C_3 + {}_4C_4 = 2^4$.

Answer First observe that ${}_4C_4$ is the number of 4 topping pizzas, ${}_4C_3$ is the number of 3 topping pizzas, ${}_4C_2$ is the number of 2 topping pizzas, ${}_4C_1$ is the number of 1 topping pizzas, and ${}_4C_0$ is the number of no topping pizzas. So the total number of pizzas is ${}_4C_0 + {}_4C_1 + {}_4C_2 + {}_4C_3 + {}_4C_4$. But we can also approach this problem differently. There are two possibilities for each topping, on the pizza or off the pizza, so the total number of pizzas is also 2^4. Thus, ${}_4C_0 + {}_4C_1 + {}_4C_2 + {}_4C_3 + {}_4C_4 = 2^4$.

15. Find the sum of the entries for the first four rows of Pascal's Triangle.

$$1+1=$$
$$1+2+1=$$
$$1+3+3+1=$$
$$1+4+6+4+1=$$

Describe what you observe. How is this related to the last problem?

Answer The sums of the entries for the nth row of Pascal's Triangle is 2^n. The last problem illustrated this for the 4th row of Pascal's Triangle.

16. Find the value of $\sum_{k=0}^{10} {}_{10}C_k$.

Answer This is the sum of the entries for the 10th row of Pascal's Triangle. Hence,

$$\sum_{k=0}^{10} {}_{10}C_k = 2^{10} = 1{,}024 .$$

17. Maya and Hadar were working on the following problem: "A coin is flipped 9 times. What is the probability that the coin will land heads 3 times and tails 6 times?" Maya said, "I think the answer is $\frac{{}_9C_3}{2^9}$." Hadar said, "I got $\frac{{}_9C_6}{2^9}$."

Who is right? Explain.

Answer They are both right. The number of ways to choose 3 heads is the same as the number of ways to choose the 6 tails, so ${}_9C_3 = {}_9C_6$.

18. Explain the idea from the last problem in terms of Pascal's Triangle.

Answer Pascal's Triangle is symmetric, so the numbers on the right side of the triangle match the numbers on the left side of the triangle.

19. Determine whether the following statement is true or false: ${}_nC_r = {}_nC_{n-r}$.

Answer True

20. Determine whether the following statement is true or false: ${}_nP_r = {}_nP_{n-r}$.

Answer False

21. Find the value of x that completes the following statement ${}_{100}C_{20} = {}_{99}C_{19} + {}_{99}C_{x}$.

Answer There are two possible answers: $x = 20$ or $x = 79$. (Make sure you are able to find both answers!

22. Find each of the following.

a. $(2x+3y)^5$

b. $(x-2)^6$

Answers a. $32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$

b. $x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64$

23. Consider the expansion of $(a+b)^{20}$. What is the coefficient of the a^6b^{14} term? ***Hint:*** You only need to find this term. Think about combination numbers rather than using Pascal's Triangle.

Answer ${}_{20}C_6 = 38,760$

24. Consider the expansion of $(x+2)^{15}$. What is the coefficient of the x^{12} term? ***Hint:*** Same type of hint as the last problem.

Answer The relevant term is $({}_{15}C_3)(x)^{12}(2)^3 = 3640x^{12}$, so the coefficient is 3,640.

25. Is the following statement true or false: $(a+b)^n = \sum_{k=0}^{n}({}_nC_k)a^{n-k}b^k$.

Answer True! This is the binomial theorem!

$(a+b)^n = ({}_nC_0)a^n + ({}_nC_1)a^{n-1}b + ({}_nC_2)a^{n-2}b^2 + ... + ({}_nC_{n-1})ab^{n-1} + ({}_nC_n)b^n$

26. **Optional Challenge Problem** The 5th row of Pascal's Triangle is 1 5 10 10 5 1 and the 7th row of Pascal's Triangle is 1 7 21 35 35 21 7 1. Notice that all of the numbers on the 5th row are divisible by 5 and all of the numbers on the 7th row are divisible by 7 (aside from the 1's on the two ends). Are there any other rows that have this property? Which ones? Explain.

27. **Optional Challenge Problem** How many odd numbers are there on the 100th row of Pascal's Triangle? What about the 200th row? How can you tell?

28. **Optional Challenge Problem** How many numbers on the 50th row of Pascal's Triangle are divisible by 50? How many numbers on the 100th row are divisible by 100? How many numbers on the 200th row are divisible by 200? How can you tell?