Algebra 2/Pre-Calculus

Name__________________

Distribution for Outcomes of Repeated Experiments (Day 6, Statistics)

In this handout, we will investigate the distribution for the outcomes of repeated experiments.

1. Suppose you flip a fair coin four times and count the number of times you get heads.

a. Fill in the table at the right. ***Hint:*** Pascal's Triangle.

# of heads	# of ways	probability
0		
1		
2		
3		
4		

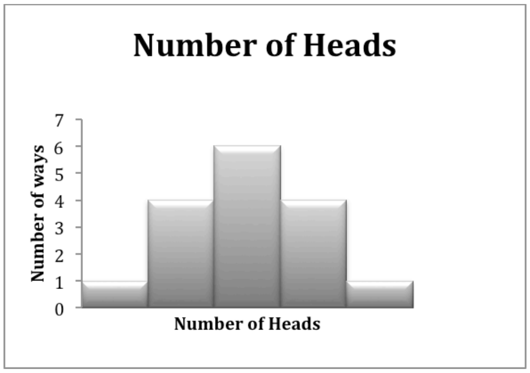
b. Make a histogram (bar graph) showing the # of heads on the x-axis and the # of ways on the y-axis. Just make a rough sketch. Don't worry about making it perfect. ***Note:*** The answers for the table in part a are provided below.

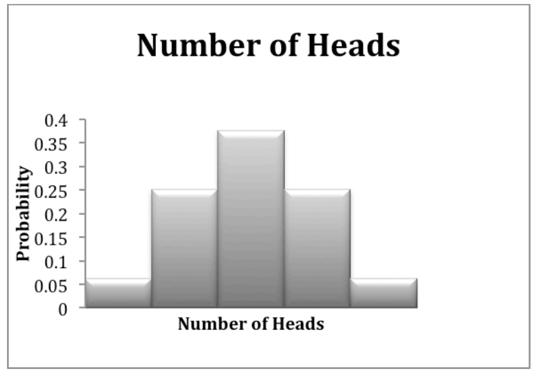
# of heads	# of ways	probability
0	1	1/16
1	4	4/16
2	6	6/16
3	4	4/16
4	1	1/16

c. Make another histogram showing the # of heads on the x-axis and the probability on the y-axis. Again, just make a rough sketch.

d. How do the shapes of the two histograms compare? What's similar? What's different?

2. Here are the histograms that you should have found in the last problem:





a. Notice that the two histograms are almost identical. What is the one key way in which they differ? ***Hint:*** What are the units for the y-axis?

b. Describe the shape of the histograms in your own words.

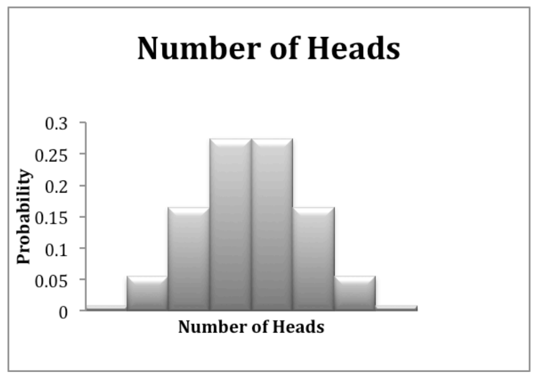
3. Suppose you flip a fair coin seven times.

a. Fill in the table at the right.

# of heads	probability
0	
1	
2	
3	
4	
5	
6	
7	

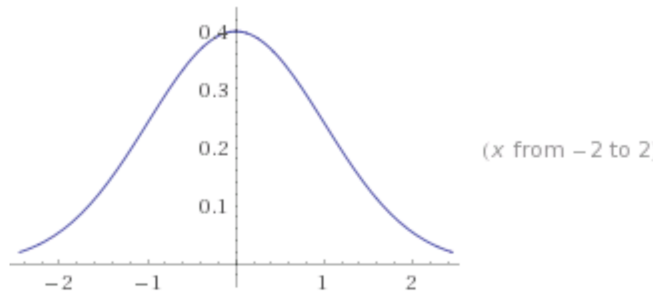
b. Make a histogram (bar graph) showing the # of heads on the x-axis and the probability on the y-axis. Again, just a rough sketch is fine.

c. How does the shape of the histogram for four flips compare with the histogram for seven flips? Are they similar? In what way? ***Note:*** The histogram that you should have found in part **b** is pictured below.



d. Without calculating the probabilities, sketch the histogram for ten flips.

You should have found that the histograms for four flips, seven flips, and ten flips all had the same basic shape. You may also have noticed that the histograms with more flips were "smoother" than the histograms for less flips. If we kept adding flips, we would approach the following shape:



This is called a normal curve. (It's also called a bell curve or a Gaussian distribution.) Its got a pretty weird equation: $y = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$, but don't worry about that for now. Our primary focus is going to be on the shape of this curve.

4. **a.** Sketch a bell curve over each of the histograms you've done thus far. You should find that the bell curve matches the ten flip histogram best (and doesn't match the four flip histograms quite as well).

b. Look at the four flip histogram. Which value (# of flips) is at the "center" of the histogram?

c. Find the mean value for four flips. (That is, find the expected value for the number of heads from four flips.)

d. How do your answers for parts **b** and **c** compare? Does the same thing happen for the seven flips? For ten flips?

Some answers b. 2 c. 2 d. The mean is at the center of the histogram

You should have found that the mean value was always at the "center" of the histogram. (Sometimes this number is actually on the histogram and sometimes it's a decimal, as in the case of seven flips, where the mean is 3.5. But in both situations, the mean is right in the middle of the histogram.)

Our next goal is to determine what sorts of values are more or less likely to occur. (You may already have some intuition about this!)

5. a. Which type of values do you think we are more likely to get: values that are close to the mean or values that are far from the mean?

b. Suppose we flip the coin seven times. What is the probability we get heads either 3 or 4 times. Remember, you've already calculated these probabilities.

c. What is the probability you get heads 5 or 6 times? (Still assuming seven flips.)

d. Are your answers to parts **b** and **c** consistent with your answer to part **a**? Explain.

e. How do your answers to parts **a** – **d** of this problem relate to this shape of the histogram? ***Hint:*** How do the values in the middle of the histogram compare to the values on the ends?

Answers a. Values close to the mean b. $\frac{35}{128}+\frac{35}{128}=\frac{70}{128}=0.547$ c. $\frac{21}{128}+\frac{7}{128}=\frac{28}{128}=0.219$

d. Yes. 5 and 6 are further from the mean and thus less likely. e. Values that are close to the mean have higher probabilities than values that are far from the mean.

In the last problem, you should have found that values near the mean were more likely than values far from the mean. We can see this from the shape of the histogram: The middle of the histogram is the highest part, but the values at the ends are very low.

So far, we have only looked at the histogram for an event with equally likely outcomes. (The fair coin has the same chance of landing heads as tails.) But what happens if we bend the coin so that it is more likely to get heads? In particular, do we get the same shape for our histogram?

6. Suppose you are flipping a bent coin. The coin is bent in such a way that it lands heads 70% of the time and lands tails 30% of the time. You flip it four times.

a. What is the probability that the coin will land heads all four times?

b. What is the probability that the coin will land HHTH (in that order)?

c. What is the probability that the coin will land HTHH (in that order)?

d. How many ways are there for the coin to land heads three times and tails once (in any order)? ***Hint:*** Think about combination numbers. There are four flips and you are choosing three of them to be heads.

e. What is the probability of getting three heads and one tail (in any order)? ***Hint:*** Use what you learned in parts **b** – **d**.

Answers a. $(0.7)^4 = 0.2401$ b. $(0.7)(0.7)(0.3)(0.7) = (0.7)^3(0.3)^1 = 0.1029$

c. $(0.7)(0.3)(0.7)(0.7) = (0.7)^3(0.3)^1 = 0.1029$ d. ${}_4C_1 = 4$ e. $4(0.7)^3(0.3)^1 = 0.4116$

6. (Continue using the coin from problem **6**)

f. How many ways are there to get two heads and two tails (in any order)?

g. What is the probability of getting two heads and two tails (in any order)? ***Hint:*** Use a calculation similar to the one you used in part **e**.

h. We will now find the probabilities for each of our remaining possibilities.

Fill in the table at the right. Notice that you have found some of these probabilities already.

# of heads	# of ways	probability
0		
1		
2		
3		
4		

i. Add up all of the number in the probability column. If they don't add up to 1, check your work. ***Caution:*** In this problem, the probability of getting 3 heads and 1 tail is different from the probability of getting 1 head and 3 tails. Why?

j. What are the similarities and differences between this table and the table for the fair coin back in problem **1**?

Some answers f. ${}_4C_2 = 6$ g. $6(0.7)^2(0.3)^2 = 0.2646$

j. Make a histogram for the unfair coin showing the # of heads on the x-axis and the probability on the y-axis. Just a rough sketch is fine.

k. Does the histogram appear to have a bell shaped distribution? (It might be a little difficult to say for sure.) Try, if possible, to sketch a bell curve on top of your histogram.

7. Now suppose we flip the unfair coin five times. (The coin still has a 70% probability landing of heads and a 30% probability of landing tails.)

a. Fill in the table at the right. Use the method that you developed in problem **5** to find the probabilities.

# of heads	# of ways	probability
0		
1		
2		
3		
4		
5		

b. Add up all of the numbers in the probability column. If they don't add up to 1, go back and check your work for part **a**.

c. Make a histogram showing the # of heads on the x-axis and the probability on the y-axis. Just a rough sketch is fine.

d. This histogram should appear roughly normal. Sketch a bell curve on top of your histogram.

8. There's a faster way to calculate the probabilities you from the last two problems. Consider the following calculation (which you could do by using the binomial theorem or by using wolframalpha):

$$(.7h+.3t)^4 = 0.2401h^4 + 0.4116h^3t + 0.2646h^2t^2 + 0.0756ht^3 + 0.0081t^4$$

a. How is this calculation related to the problem of flipping the unfair coin four times? Why does this work?

b. Use this trick to find the probabilities for flipping the unfair coin six times. (You may use wolframalpha to do the calculations.) Then sketch the histogram for this situation. Does the histogram appear to be normal?

9. Earlier in this handout, we found that values close to the mean are much more likely than values far from the mean. (This idea was explored in question **4**.) But so far we've only tested this for situations where outcomes were *equally likely* (the fair coin). In this question, we investigate whether the same thing happens with the unfair coin.

a. Find the mean number of heads for a single flip of the unfair coin. (Remember, this is an expected value calculation.)

b. What is the mean number of heads from four flips of the unfair coin? (Remember, there's a fast way to do this.) What's the mean number of heads from five flips? From six flips?

c. In the case of the fair coin, the mean was at the center of the histogram. Because of the shape of the bell curve, we found that values close to the mean were much more likely than values far from the mean. Did the same thing happen for the unfair coin? Is the mean still at the center of a bell shaped distribution?