Algebra 2/Pre-Calculus

Name_____

Dice Rolling and Polynomial Power (Day 1, Statistics)

The goal of this handout is to explore the connections between die rolling, number spinners, and polynomial multiplication.

1. Suppose you roll two standard six sided dice.

a. What is the probability that the sum of the two numbers will be a 12?

b. What is the probability that the sum will be an 11?

c. What is the probability that the sum will be a 9?

Answers a. $\frac{1}{36}$ b. $\frac{2}{36}$ c. $\frac{4}{36}$

2. A table can be very helpful when answering questions of the type in problem **1**. Make a table representing the sum you get when you roll two six sided dice. ***Note:*** If you're not sure how to do this, the answer is provided on the next page. But try it on your own first!

3. Here's a table representing all of the possible sums you could get if you rolled two six sided dice:

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

a. How many ways are there to get a sum of 6?

b. What is the probability that the sum is a 7?

c. What is the probability that the sum is either a 5 or a 6?

d. What is the probability that the sum is at least a 9?

e. What is the probability that the sum is less than 4?

f. What is the probability that the sum is divisible by 3?

g. What is the probability that the sum is a prime number?

h. What is the probability that the sum is greater than 0?

i. What is the probability that the sum is greater than 12?

Answers a. 5 b. $\frac{6}{36}$ c. $\frac{9}{36}$ d. $\frac{10}{36}$ e. $\frac{3}{36}$ f. $\frac{12}{36}$ g. $\frac{15}{36}$ h. 1 i. 0

4. Suppose you roll two six sided dice, but they don't have the usual numbers. The first die has the following numbers {3, 4, 4, 4, 5, 5} and the second die has the numbers {2, 2, 2, 6, 6, 6}.

a. Complete the table showing the possible sums you could get from these dice. ***Note:*** If you're not sure how to do this, you can see the completed table on the next page. But try it on your own first!

	3	**4**	**4**	**4**	**5**	**5**
2						
2						
2						
6						
6						
6						

b. What is the probability that the sum is a 6?

c. What is the probability that the sum is a 5?

d. What is the probability that the sum is 4?

e. What is the probability that the sum is between 8 and 12?

f. Does the table above look a little bit like a multiplication table? Explain. ***Note:*** This question is purposely open-ended. It is connected to an idea that we will explore on the next page. But see what you observe on your own first.

Some answers b. $\frac{9}{36}$ c. $\frac{3}{36}$ d. 0 e. $\frac{18}{36}$

5. Here the table from the last problem again, this time with the sums filled in.

	3	**4**	**4**	**4**	**5**	**5**
2	5	6	6	6	7	7
2	5	6	6	6	7	7
2	5	6	6	6	7	7
6	9	10	10	10	11	11
6	9	10	10	10	11	11
6	9	10	10	10	11	11

Since many of the numbers are repeated, we can summarize the table in the following way:

	one 3	**three 4's**	**two 5's**
three 2's	three 5's	nine 6's	six 7's
three 6's	three 9's	nine 10's	six 11's

It's almost like we're "multiplying" the columns and the rows, in some sense. Let's do a related multiplication: $(3x^2 + 3x^6)(x^3 + 3x^4 + 2x^5)$.

a. Find the product: $(3x^2 + 3x^6)(x^3 + 3x^4 + 2x^5)$

b. Notice that we can illustrate this multiplication with the following table:

	x^3	$3x^4$	$2x^5$
$3x^2$	$3x^5$	$9x^6$	$6x^7$
$3x^6$	$3x^9$	$9x^{10}$	$6x^{11}$

Now explain how the multiplication $(3x^2 + 3x^6)(x^3 + 3x^4 + 2x^5)$ is related to the problem with the dice.

6. Let's take a closer look at the product from the last problem:

$$(3x^2 + 3x^6)(x^3 + 3x^4 + 2x^5) = 3x^5 + 9x^6 + 6x^7 + 3x^9 + 9x^{10} + 6x^{11}$$

We say that this product is "associated" with the rolling of the dice. For example, we can see quickly that there are 6 ways for the sum to be an 11 because one of the terms in our answer was $6x^{11}$.

In the rest of this problem, we will look at more examples connecting dice rolling with multiplication.

a. Consider the multiplication $(4x^5 + 2x^7)(2x^2 + 2x^4 + 2x^6)$. How is this multiplication related to rolling two six sided dice? What are the numbers on each of the dice?

b. Find the product of $(4x^5 + 2x^7)(2x^2 + 2x^4 + 2x^6)$.

c. In part **a**, you should have found that the two dice had the following numbers: {5, 5, 5, 5, 7, 7} and {2, 2, 4, 4, 6, 6}. How many ways are there for the sum to be 7?

d. How many ways are there for the sum to be 9?

e. What is the probability that the sum is 9?

Some answers b. $8x^7 + 12x^9 + 12x^{11} + 4x^{13}$ c. 8 d. 12 e. $\frac{12}{36}$

7. Suppose you roll two eight sided dice with the following numbers:

{2, 2, 2, 4, 4, 4, 5, 5} and {3, 3, 3, 3, 3, 8, 8, 8}.

a. What multiplication problem is associated with these dice?

b. Do the multiplication problem you wrote down in the last problem. (In other words, find the product.)

c. How many ways are there to get a sum of 7?

d. What is the probability that the sum is 7? ***Hint:*** When we were rolling two six sided dice, the probabilities had a 36 in the denominator. What should the denominator be when we are rolling two eight sided dice?

e. What is the probability that the sum is 13?

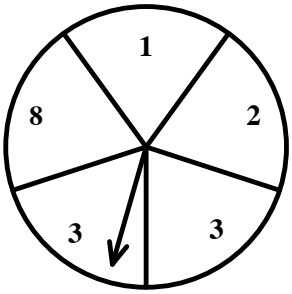
f. Find the sum of the coefficients for the product you found in part **b**. Why does this make sense?

Answers a. $(3x^2 + 3x^4 + 2x^5)(5x^3 + 3x^8)$ b. $15x^5 + 15x^7 + 10x^8 + 9x^{10} + 9x^{12} + 6x^{13}$ c. 15 d. $\frac{15}{64}$ e. $\frac{6}{64}$ f. $15+15+10+9+9+6=64$ (There are 64 total possibilities, counting repeats.)

8. Imagine a toy spinner with the numbers with the numbers 1, 2, 3, 3, and 8. Printed on it. When the arrow is spun, it is equally likely to land on any of the five numbers. (The spinner is pictured below.)

a. Suppose you spin it twice. What is the polynomial multiplication associated with this problem?

b. You should have found that the answer to the last question was $(x^1 + x^2 + 2x^3 + x^8)(x^1 + x^2 + 2x^3 + x^8)$. Do this multiplication.



c. How many ways are there to get a six for the sum? How many ways could the sum be a four? How many ways could the sum be a sixteen? ***Hint:*** Use your answer from the last problem.

d. What is the probability that the sum will be a six? What is the probability that the sum with be a four? What is the probability that the sum will be a sixteen? ***Hint:*** Think about how you got the denominator in the problems involving the dice.

e. What is the sum of the coefficients for the product you found in part **b**?

Answers b. $x^2 + 2x^3 + 5x^4 + 4x^5 + 4x^6 + 2x^9 + 2x^{10} + 4x^{11} + x^{16}$ c. 4 ways to get 6, 5 ways to get 4, 1 way to get 16 d. $\frac{4}{25}$, $\frac{5}{25}$, $\frac{1}{25}$ e. 25

9. Let's continue using the number spinner with the numbers {1, 2, 3, 3, 8}. (Same as the last problem.) This time, we spin it four times.

a. What is the polynomial multiplication associated with this problem?

Hint: When we spun it twice, we did the multiplication $(x^1 + x^2 + 2x^3 + x^8)^2$.

b. You should have found that the multiplication was $(x^1 + x^2 + 2x^3 + x^8)^4$. It would be unreasonable for me to ask you to do this multiplication by hand, so we will use a free online tool to do the multiplication by hand.

Go to www.wolframalpha.com, then type in (x^1+x^2+2x^3+x^8)^4. Press RETURN. The program will give you lots of stuff (graphs, etc.) but the thing we want is the "Expanded form." Copy what you get for this in the space below.

c. How many ways are there for the sum to be 17?

d. What is the probability that the sum will be 20?

e. Which sum is most likely to occur? What is the probability of getting this sum?

f. What is the probability that the sum is at least 25?

g. What is the sum of the coefficients for the product you found in part **b**? ***Hint:*** There is a fast way to find the sum!

Some answers c. 32 d. $\frac{30}{5^4} = \frac{30}{625}$ e. The sum with the highest probability is 15. The probability is $\frac{72}{625}$. f. $\frac{17}{625}$ g. $5^4 = 625$

10. The numbers on a standard six-sided die are {1, 2, 3, 4, 5, 6}. Suppose you roll this die three times.

a. What is the associated multiplication?

b. Use wolframalpha to do this multiplication. Write your answer in the space below.

c. What is the probability that the sum will be either a 13 or a 14?

d. What is the probability that the sum will be at least 15?

e. What is the probability that the sum will be less than 15?

f. What is the probability that the sum will be at most 6?

g. Parts **d** and **f** had the same answer. Why did this happen?

Some answers a. $(x + x^2 + x^3 + x^4 + x^5 + x^6)^3$ c. $\frac{21}{216} + \frac{15}{216} = \frac{36}{216}$

d. $\frac{10}{216} + \frac{6}{216} + \frac{3}{216} + \frac{1}{216} = \frac{20}{216}$ e. $1 - \frac{20}{216} = \frac{196}{216}$ f. $\frac{10}{216} + \frac{6}{216} + \frac{3}{216} + \frac{1}{216} = \frac{20}{216}$

11. Suppose you have a number spinner with the numbers {1, 1, 4, 4, 4, 5}.

a. Suppose you spin it twice. What is the probability that the sum will be a 5? ***Hint:*** Start by doing the associated polynomial multiplication. Do this multiplication by hand.

b. Suppose you spin it twice. What is the probability that the sum will be at least 9?

c. Suppose you spin it three times. What is the probability that the sum will be 12? ***Note:*** Use wolframalpha for the polynomial multiplication.

d. Suppose you spin it four times. What is the probability that the sum will be less than 10?

Answers a. $\frac{12}{36}$ b. $\frac{7}{36}$ c. $\frac{27}{216}$ d. $\frac{144}{1296}$

12. Suppose you have a number spinner with just two numbers: 1 and 0.

a. Suppose you spin it twice. What is the probability that the sum will be a 2? ***Hint:*** Start by doing the associated polynomial multiplication. Do this multiplication by hand.

b. You should have found that the associated polynomial multiplication was $(x^1 + x^0)^2$. Notice that this is the same as $(x+1)^2$. Now suppose we spin the spinner three times. What is the probability that the sum will be a 1? ***Note:*** Do this calculation without using wolframalpha.

c. You should have found that $(x+1)^3 = x^3 + 3x^2 + 3x + 1$. Now suppose we spin it five times. What is the probability that the sum will be a 3? Do this problem without using wolframalpha. ***Hint:*** What's the fast way of finding $(x+1)^5$?

d. Now suppose we spin it 15 times. What is the probability that the sum will be an 8? Do this problem without using wolframalpha. ***Hint:*** Do you need to find every term of $(x+1)^{15}$?

Answers a. $\frac{1}{4}$ b. $\frac{3}{8}$ c. $\frac{10}{32}$ d. $\frac{6435}{32768}$