Algebra 2/Pre-Calculus

Name_____

Applications of Normal Distributions (Day 10, Statistics)

In this handout, we will use the bell curve to approximate probabilities in a wide range of situations.

For all probability problems on this handout, use the normal curve to approximate the probabilities.

1. Consider the following question: "Suppose you flipped a fair coin 100 times. What is the probability of getting at least 60 heads?" (We will answer this question in part **j** of this problem.)

a. Before doing any calculations, make a guess for what you think the probability will be.

b. Find the mean and standard deviation for the number of heads from one hundred flips of the coin.

c. Which values are within one standard deviation of the mean?

d. We can estimate the probability of getting one of these values by using the normal curve: $y = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$. Graph the normal curve on your calculator, then use the procedure from the last handout to find the probability.

(Press 2nd CALC, choose 7: $\int f(x)dx$, enter -1 as the lower bound and enter 1 as the upper bound.) ***Note:*** This is called the **integral** of the function. It finds the area under the curve, which we use to determine the probability.

Some answers b. mean = 50, standard deviation = 5 c. 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55 d. 0.6827 = 68.27%

e. Make a sketch of the bell curve. Shade in the area between one standard deviation below the mean and one standard deviation above the mean. ***Note:*** This is the picture you should have gotten from your calculator in part **d**.

f. How many standard deviations are there between 50 heads and 60 heads?

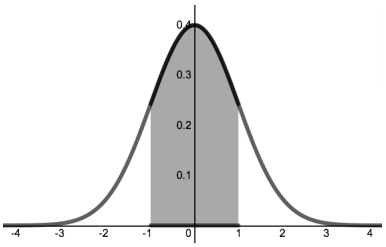
g. Use the normal curve to estimate the probability of getting between 50 and 60 heads.

Hint: What are the lower and upper bounds you should use?

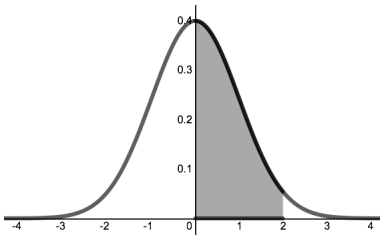
h. Sketch the bell curve. Shade the area between the mean and two standard deviation above the mean. ***Note:*** To see this on your calculator without the shading from the previous problem, turn off the equation and turn it back on before using the integral function.

Answers f. 2 g. 0.4772 = 47.72% (use 0 as the lower bound and 2 as the upper bound)

e.



h.

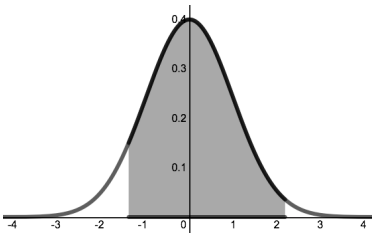
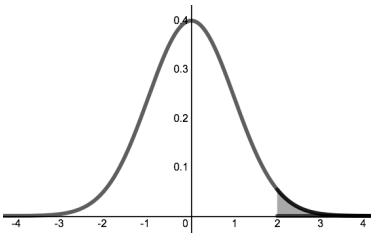
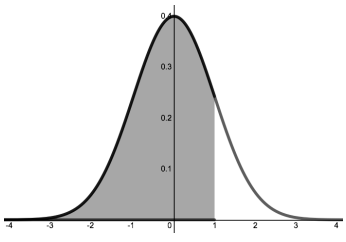


i. Use the normal curve to estimate the probability that the number of heads is between 43 and 61. Then sketch a picture of the normal curve and shade in the appropriate area. ***Hint:*** How many standard deviations below the mean is 43? How many standard deviations above the mean is 61? These numbers will be your lower and upper bounds.

j. You should have found that the probability was 0.9053. (Lower bound: $\frac{43-50}{5} = -1.4$, Upper bound: $\frac{61-50}{5} = 2.2$) This means that 90.53% of the time, the number of heads is between 43 and 61. Now find the probability of getting at least 60 heads. Then sketch a picture of the normal curve and shade in the appropriate area. ***Note:*** Our lower bound is 2 because the value of 60 is 2 standard deviations above the mean. But what should we put for the upper bound? The normal curve goes on forever, but it gets negligibly small after about 5 standard deviations. So we will use 2 as our lower bound and 5 as our upper bound.

k. You should have found that the probability was 0.0227. This means we would only get 60 or more heads about 2.27% of the time. (How did this answer compare to your guess in part **a**?) Now find the probability of getting 55 or fewer heads. Again, sketch a picture of the normal curve and shade in the appropriate area.

Answers

i. Probability = 90.53%	j. Probability = 2.27%	k. Probability = 84.13%
		

2. Suppose you're playing a game where you roll a standard six-sided die. If you roll a 6, you win. If you roll any other number, you lose. You play the game 300 times

a. What are the mean and standard deviation for the number of wins?

b. Use the normal curve to estimate the probability that the number of wins is between 40 and 60.

c. Use the normal curve to estimate the probability that the number of wins is at least 65.

d. You friend tells you that he also played this game 300 times and won 90 times. How believable is your friend's claim? Suppose your conclusion with a calculation.

Answers a. mean = 50, standard deviation = 6.455 b. 87.87% (lower bound = -1.549, upper bound = 1.549) c. 1.01% (lower bound = 2.3238, upper bound = 5) d. Extremely unlikely.

3. **a.** On the normal curve, what is the probability of getting a result that is within 1 standard deviation of the mean? Within two standard deviations of the mean? Within three standard deviations of the mean?

b. Statisticians often have the values 68-95-99.7 memorized and will sometimes refer to the "68-95-99.7 rule." Explain what this rule is.