

Algebra 2/Pre-Calculus

More Practice (Day 5, Logarithmic Functions)

Name _____

Key Properties, Definitions, Etc.

Definition: $\log_b M$ = The answer to the question “ b to what power equals M ?”

Definition: $\log M$ is the same as $\log_{10} M$

Calculator Usage: You should be able to evaluate logs of any base on your calculator.

Property: We can only take the log of a positive number. (Make sure you can explain why we can't take the log of zero or a negative number.)

Property: We can switch between “log form” and “exponent form” as demonstrated in the following examples: $\log_2 8 = 3 \longleftrightarrow 2^3 = 8$, $\log_b X = Y \longleftrightarrow b^Y = X$,

$$\log_5(x + 4) = 2 \longleftrightarrow 5^2 = x + 4.$$

Definition: An “extraneous solution” is an incorrect answer we get despite doing all correct steps. For example, the equation $\log(2 - x) = \log(9 - 2x)$ has no actual solutions, but it has an extraneous solution of $x = 7$. **Remember:** We are only allowed to take the log of positive numbers.

Properties based on exponent rules: Each of the rules of exponents can be expressed in terms of logs as follows.

Exponent Rules

$$b^x \cdot b^y = b^{x+y} \longleftrightarrow$$

$$\frac{b^y}{b^x} = b^{x-y} \longleftrightarrow$$

$$(b^x)^a = b^{ax} \longleftrightarrow$$

Log Rules

$$\log_b MN = \log_b M + \log_b N$$

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

$$\log_b M^a = a \log_b M$$

Make sure you can explain why each of these log rules work.

Review Problems

1. Estimate the value of each of the following logarithms without using your calculator.

a. $\log_3 34$

b. $\log_2 15$

c. $\log 137$

d. $\log_2\left(\frac{1}{7}\right)$

2. Evaluate each of the following logarithms without using your calculator.

a. $\log_2 16$

b. $\log_5 \sqrt[3]{5}$

c. $\log \frac{1}{1000}$

d. $\log_6 6^{10}$

e. $\log_3(-9)$

f. $\log_{11}(1)$

g. $\log(0)$

h. $3 \log_7(49) + 8$

i. $5 - \log_5\left(\frac{1}{25}\right)$

j. $\log_6(3) + \log_6(12)$

k. $\log(5000) - \log(5)$

l. $\log_2 32^7$

m. $\log_4 8$

n. $\log_2\left(\frac{\sqrt{2}}{4}\right)$

o. $\log_9 \sqrt[4]{3}$

Answers a. 4 b. $\frac{1}{3}$ c. -3 d. 10 e. undefined f. 0 g. undefined h. 14 i. 7 j. 2

k. 3 l. 35 m. $\frac{3}{2}$ n. $-\frac{3}{2}$ o. $\frac{1}{8}$

3. Suppose you are given the following approximations: $\log_7 3 = 0.6$, $\log_7 5 = 0.8$, and $\log_7 12 = 1.3$. Find each of the following **without** using your calculator.

a. $\log_7 15$ *Hint:* $15 = 3 \cdot 5$

b. $\log_7 60$

c. $\log_7 \frac{5}{12}$

d. $\log_7 \frac{1}{4}$

e. $\log_7 9$

f. $\log_7 12^{20}$

Parts **g** and **h** and optional challenge problems.

g. $\log_7 100$

h. $\log_7(3.75)$

Answers: a. 1.4 b. 2.1 c. -0.5 d. -0.7 e. 1.2 f. 26 g. 2.3 h. 0.7

4. Solve each of the following equations **without** using your calculator. **Caution:** Some of these equations have extraneous solutions.

a. $4\log_2(x - 3) + 1 = 13$

b. $8 = 5 - 6\log_5 x$

c. $\log x + \log 5 = 2$

d. $\log_6(x - 5) + \log_6(x + 4) = 2$

e. $\log_2(x^2 + 25) = 5$

f. $\log_7 21 = \log_7(3x - 9)$

g. $\log(2x + 5) = \log(7x - 30)$

h. $\log_8(x - 7) = \log_8(3x - 11)$

i. $\log_6 4 + 2\log_6(x + 5) = 2$

j. $2\log_2 x + \log_2(x^2 - 6) = 4$

Answers a. 11 b. $\frac{1}{\sqrt{5}}$ c. 20 d. 8 only e. $\pm\sqrt{7}$ (both work) f. 10 g. 7

h. no solution (2 is extraneous) i. -2 only j. $\sqrt{8}$ only

5. Solve each of the following equations. You may use the log features of your calculator but not the graphing features. Show your work. Final answers should be rounded to two decimal places.

a. $3 \cdot 2^x - 7 = 143$

b. $10 = 1.34^{2x-3}$

Answers a. 5.64 b. 5.43

- d. Nina invests some money. After 11 years, her investment has doubled. How many years does it take for her investment to triple? *Hint:* Start by finding growth rate as a percent.

Answers a. 6.50 years b. 4.90 years c. Uma (make sure you can show why!) d. 17.43 years