Algebra 2/Pre-Calculus

Name__________________

Long Division (Day 6, Polynomials)

In this handout, we will learn how to do polynomial long division. We will also explore how we can use long division to better understand polynomial factoring.

1. Do the following problems by long division. (Yes, the same long division that you did in elementary school!) If the answer involves a remainder, include the remainder in the form of a fraction. ***Note:*** If you don't remember how to do this, the solutions are provided below. But try it own your own first!

a. $12\overline{)2772}$

b. $12\overline{)2779}$

Answers and Solutions

a. 231

$$\begin{array}{r} 231 \\ 12\overline{)2772} \\ \underline{24} \\ 37 \\ \underline{36} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

b. $231 + \frac{7}{12}$

$$\begin{array}{r} 231\tfrac{7}{12} \\ 12\overline{)2779} \\ \underline{24} \\ 37 \\ \underline{36} \\ 12 \\ \underline{12} \\ 7 \end{array}$$

Terms and Vocabulary When we do the division $\frac{2779}{12} = 231 + \frac{7}{12}$, we say that 231 is the quotient, 7 is the remainder, 12 is the divisor, and 2779 is the dividend.

2. Polynomial long division is done in the same way that we do long division with numbers. Attempt the following two polynomial divisions. ***Note:*** Use the two long divisions you just did as a model for the following polynomial divisions. (If you get stuck, look ahead to the solutions provided below.)

a. $x+2\overline{)2x^3+7x^2+7x+2}$

b. $x+2\overline{)2x^3+7x^2+7x+9}$

Answers and Solutions

a. $2x^2+3x+1$

$$\begin{array}{r} 2x^2+3x+1 \\ x+2\overline{)2x^3+7x^2+7x+2} \\ \underline{2x^3+4x^2} \\ 3x^2+7x \\ \underline{3x^2+6x} \\ x+2 \\ \underline{x+2} \\ 0 \end{array}$$

b. $2x^2+3x+1+\frac{7}{x+2}$

$$\begin{array}{r} 2x^2+3x+1+\frac{7}{x+2} \\ x+2\overline{)2x^3+7x^2+7x+9} \\ \underline{2x^3+4x^2} \\ 3x^2+7x \\ \underline{3x^2+6x} \\ x+2 \\ \underline{x+2} \\ 7 \end{array}$$

3. Long division for polynomials is a little trickier than long division for whole numbers because polynomials sometimes involve minus signs. Do the following two long divisions. (The solutions are provided below.) ***Note:*** Be very careful with you minus signs!

a. $x-4\overline{\smash{)}x^3-x^2-14x+13}$

b. $x+2\overline{\smash{)}-2x^4-7x^3-2x^2+7x-9}$

Answers and Solutions

a. $x^2+3x-2+\frac{5}{x-4}$

$$
\begin{array}{r}
x^2+3x-2+\frac{5}{x-4} \\
x-4\overline{\smash{)}x^3-x^2-14x+13} \\
\underline{x^3-4x^2} \qquad\qquad\quad \\
3x^2-14x \qquad\ \ \\
\underline{3x^2-12x} \qquad\ \ \\
-2x+13 \\
\underline{-2x+8} \\
5
\end{array}
$$

b. $-2x^3-3x^2+4x-1+\frac{-7}{x+2}$

$$
\begin{array}{r}
-2x^3-3x^2+4x-1+\frac{-7}{x+2} \\
x+2\overline{\smash{)}-2x^4-7x^3-2x^2+7x-9} \\
\underline{-2x^4-4x^3} \qquad\qquad\qquad\quad \\
-3x^3-2x^2 \qquad\qquad \\
\underline{-3x^2-6x^2} \qquad\qquad \\
4x^2+7x \qquad \\
\underline{4x^2+8x} \qquad \\
-x-9 \\
\underline{-x-2} \\
-7
\end{array}
$$

Observation: We need to be super careful when subtracting with negatives. For example, if you look at the solution for the first division problem on this page, it appears to say $-x^2-4x^2=3x^2$, which is clearly untrue. But we have to remember that what we are subtracting the $-4x^2$. The calculation we are actually doing is $-x^2-(-4x^2)=3x^2$.

4. Here are some division problems where we divide by quantities that aren't linear. As in the previous problems, solutions are provided below, but try them on your own first.

a. $x^2+3x+4\overline{)x^4+8x^3+20x^2+25x+10}$ **b.** $3x^2-2x-5\overline{)6x^3+5x^2-17x-11}$

Answers and Solutions

a. $x^2+5x+1+\frac{2x+6}{x^2+3x+4}$

$$\begin{array}{r} x^2+5x+1+\frac{2x+6}{x^2+3x+4} \\ x^2+3x+4\overline{)x^4+8x^3+20x^2+25x+10} \\ \underline{x^4+3x^3+4x^2\qquad\qquad\quad} \\ 5x^3+16x^2+25x\qquad \\ \underline{5x^3+15x^2+20x}\qquad \\ x^2+5x+10 \\ \underline{x^2+3x+4} \\ 2x+6 \end{array}$$

b. $2x+3+\frac{-x+4}{3x^2-2x-5}$

$$\begin{array}{r} 2x+3+\frac{-x+4}{3x^2-2x-5} \\ 3x^2-2x-5\overline{)6x^3+5x^2-17x-11} \\ \underline{6x^3-4x^2-10x}\qquad \\ 9x^2-7x-11 \\ \underline{9x^2-6x-15} \\ -x+4 \end{array}$$

5. Suppose we are trying to do the following division: $\dfrac{x^3 - 7x + 4}{x - 3}$

a. What's problematic about setting up the division as $x - 3\overline{)x^3 - 7x + 4}$? What would be a better way of setting it up?

b. It's a lot easier to keep track of our work if we set it up in the following way: $x - 3\overline{)x^3 + 0x^2 - 7x + 4}$. Now do this division.

c. We can use www.wolframalpha.com to check our answers. Go to this site on your computer or smart phone and plug in (x^3-7x+4)/(x-3). Press ENTER and scroll down to "Alternate forms." Do you see the answer you got in part **b**?

d. Perform the division $\dfrac{x^3 - 27}{x - 3}$. Use wolframalpha to check your answer. ***Hint:*** Think about how you set up the long division on the last problem.

e. Factor $x^3 - 27$. ***Hint:*** Think about the answer you got in part **c**.

6. In this problem, we will explore the relationship between division and factoring.

a. Is 8 a factor of 48? Is 8 a factor of 49? Explain how you know.

b. 8 is a factor of 48 because $\frac{48}{8} = 6$. Conversely, when we do the division $\frac{49}{8}$ we get a remainder, so 8 is not a factor of 49. Is $x+4$ a factor of $x^3 - 5x^2 - 22x + 56$? How do you know?

c. If you performed the division $\dfrac{x^3 - 5x^2 - 22x + 56}{x+4}$, you found that there was no remainder. Can you completely factor $x^3 - 5x^2 - 22x + 56$?

d. Since $\dfrac{x^3 - 5x^2 - 22x + 56}{x+4} = x^2 - 9x + 14$, we can conclude that $x^3 - 5x^2 - 22x + 56 = (x+4)(x^2 - 9x + 14) = (x+4)(x-2)(x-7)$. Is $x-3$ a factor of $x^3 + 4x^2 - 18x - 7$? Explain how you know.

e. When we do the division $\frac{x^3+4x^2-18x-7}{x-3}$, we get a remainder of 2. Therefore, $x-3$ is not a factor of $x^3+4x^2-18x-7$. Is x^2-x-1 a factor of $x^4-x^3-10x^2+9x+9$?

f. You should have found that x^2-x-1 was a factor. Now, factor $x^4-x^3-10x^2+9x+9$ as much as possible.

Answer f. $x^4-x^3-10x^2+9x+9=(x-3)(x+3)(x^2-x-1)$