Algebra 2/Pre-Calculus

Name__________________

Introduction to Exponential Functions (Day 1, Exponential Functions)

In this handout, we will introduce exponential functions.

Definition We say $f(x)$ is an *exponential function* if $f(x) = ab^x$ where $a \neq 0$, $b > 0$, and $b \neq 1$.

Some Examples of Exponential Functions:

$f(x) = 2^x$, $f(x) = 4 \cdot 5^x$, $f(x) = -10^x$, $f(x) = 2000(1.02)^x$, $f(x) = 500(0.98)^x$

Some Examples of Functions involving Exponents that are <u>not</u> Exponential Functions:

$f(x) = x^2$, $f(x) = 7x^{10}$, $f(x) = (-2)^x$, $f(x) = 3 \cdot 2^x + 8$

1. Write three examples of exponential functions.

2. When we defined the exponential function $f(x) = ab^x$, we specified that $a \neq 0$. Why do we do this?

3. We also specified that $b > 0$ and $b \neq 1$. Why do we do this?

4. The goal of this problem is to explore y-intercepts for exponential functions.

a. Find the y-intercept for the function $f(x) = 3 \cdot 4^x$. ***Hint:*** How do we find the y-intercept for any function?

b. Find the y-intercept for the function $f(x) = -6\left(\frac{3}{7}\right)^x$.

c. Find the y-intercept for the function $f(x) = ab^x$

5. In this problem, we will explore a key property for exponential functions.

a. Suppose $f(x) = 3 \cdot 2^x$. Fill in the table below:

x	***f*(*x*)**
0	
1	
2	
3	
4	
5	

b. Look at the table you just created. When you add 1 to the input, what happens to the output?

c. How is $f(x+1)$ related to $f(x)$? ***Hint:*** This is related to what you found in part **b**.

d. You should have found that $f(x+1) = 2 \cdot f(x)$. (In other words, every time we add 1 to our input, we multiply our output by 2.) Now suppose $f(x) = ab^x$. How is $f(x+1)$ related to $f(x)$?

e. You should have found that $f(x+1) = b \cdot f(x)$. (In other words, every time we add 1 to our input, we multiply our output by b.) This is sometimes referred to as the "add-multiply property" and it is a key feature of exponential functions.

How is $f(x+n)$ related to $f(x)$?

6. Here's a function that involves exponents: $g(x) = 5 \cdot 3^x + 4$.

a. Is this an exponential function? Explain why or why not.

b. According to our definition, this is not an exponential function. Why did we define exponential functions as $f(x) = ab^x$ rather than $f(x) = ab^x + c$? ***Hint:*** Does $g(x) = 5 \cdot 3^x + 4$ have the "add-multiply property" we found in problem **5d**?

Exponential functions are important in modeling a wide variety of real world phenomena, which we will explore in the following problems.

7. After graduating college, Emily gets a job with a salary of \$35,000. Furthermore, the company guarantees Emily that for every year she works, she will receive a 7% raise.

a. Fill in the following table.

years	***salary***
0	\$35,000
1	
2	
3	
4	
5	

b. You should have found that Emily's salary after 1 year was \$37,450. One way to do this calculation is to find 7% of \$35,000 and add it to the original \$35,000, as shown below:

$$35{,}000(0.07) + 35{,}000 = 37{,}450$$

Is there another, faster way to do this calculation? Could you do it with a single multiplication?

c. Here's a faster way to do the calculation: $35{,}000(1.07) = 37{,}450$. Why does this give us the same value for Emily's salary after 1 year?

d. How could you use Emily's year 1 salary of \$37,450 to find Emily's year 2 salary? Could we do this with a single multiplication?

e. You should have found that Emily's year 2 salary was \$40,071.50 because $37450(1.07) = 40071.50$. Suppose $f(x)$ is a function that gives Emily's salary after x years. (So $f(0) = 35000$, $f(1) = 37450$, $f(2) = 40071.50$, etc.) Find a formula for $f(x)$. ***Note:*** This question is really important. Make sure you have answered it successfully before you move on.

f. You should have found that $f(x) = 35000(1.07)^x$. What will Emily's salary be after 10 years? After 20 years?

g. Suppose Emily's company offered her an alternate compensation plan in which she was offered a raise of \$5,000 every year. (Her starting salary is still \$35,000.) How does this compensation plan compare to the original plan? Is one plan better than the other? Explain.

h. Suppose $g(x)$ is a function that gives Emily's salary after x years on the alternate plan. (So $g(0) = 35000$, $g(1) = 40000$, $g(2) = 45000$, etc.) Find a formula for $g(x)$. How does this formula compare with your formula for $f(x)$? In what ways are they similar? In what ways are they different?

8. There are currently 2000 rabbits in Carrot County, Montana. The population grows by 20% each year.

a. How many rabbits will there be in Carrot County one year from now?

b. How many rabbits will there be in Carrot County 10 years from now?

c. You should have found that there were 2400 rabbits after one year and 12383 rabbits after 10 years. Suppose $p(x)$ is a function that gives the rabbit population after x years. (So $p(0) = 2000$, $p(1) = 2400$, etc.) Find a formula for $p(x)$.

d. You should have found that $p(x) = 2000(1.20)^x$. What was the rabbit population four years *ago*? ***Hint:*** What sort of exponent should we use to represent "four years *ago*?"

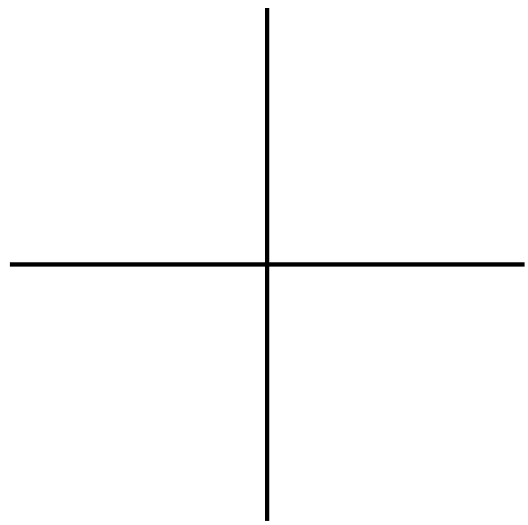
Answer: 965 rabbits. Make sure this is what you got.

e. **(Optional Challenge)** Here's a common mistake students make doing the last problem. "Every year, the rabbit population goes up by 20%. But the question asks for years ago, so we should subtract off 20%. Therefore, the population four years ago should be $2000(0.80)^4 = 819$."

What's wrong with this reasoning?

f. Complete the following table. Then make a sketch of the graph.

years	*population*
-8	
-6	
-4	
-2	
0	2000
2	
4	
6	
8	



g. Check your graph by graphing on the calculator. Sketch your graph below. ***Caution:*** Make sure to choose your window carefully!

h. What happens to the population over time? What happens on the right side of the graph? What happens on the left side of the graph?

9. Thomas buys a boat for $15,000. Unfortunately, the boat loses 12% of its value every year.

a. What percent of its value does the boat retain each year?

b. What is the value of the boat after one year?

c. What is the value of the boat after five years?

d. Suppose $f(x)$ represents the value of the boat after x years. Find a formula for $f(x)$.

e. You should have found that $f(x) = 15000(0.88)^x$. Graph this function on your calculator and sketch the graph below. ***Be careful:*** Make sure you choose a good viewing window.

f. How many years will it take for the value of the boat to reach $6000. ***Note:*** You will need to use the graphing features on your calculator to solve this problem. If you need a hint, look ahead to part **g**. But try to figure it out on your own first.

g. Here's a way of solving the last problem. On your calculator, graph $y_1 = 15000(0.88)^x$ and $y_2 = 6000$. Then press 2nd CALC and scroll down to "Intersect." Then press enter three times (first curve, second curve, guess). The answer will appear below.

Answer 7.17 years

9. (Optional Challenge) We defined exponential functions to have the form $f(x) = ab^x$. Some exponential functions can be "disguised" in forms that look different.

a. Is the function $f(x) = 3(2)^{x+2}$ an exponential function? If so, rewrite it in the form $f(x) = ab^x$.

b. Is the function $f(x) = 3(2)^{4x}$ an exponential function? If so, rewrite it in the form $f(x) = ab^x$.

c. What about the function $f(x) = 3(2)^{4x+2}$? If it is an exponential function, rewrite it in the form $f(x) = ab^x$.