Algebra 2/Pre-Calculus

Name__________________

Graphing (Day 4, Polynomials)

In this handout, we will learn to graph polynomials by hand. In particular, we are interested in making sketches of polynomial graphs that accurately show x-intercepts, y-intercept, multiplicity of roots, and end behavior. (We'll explain what these terms mean in this handout.)

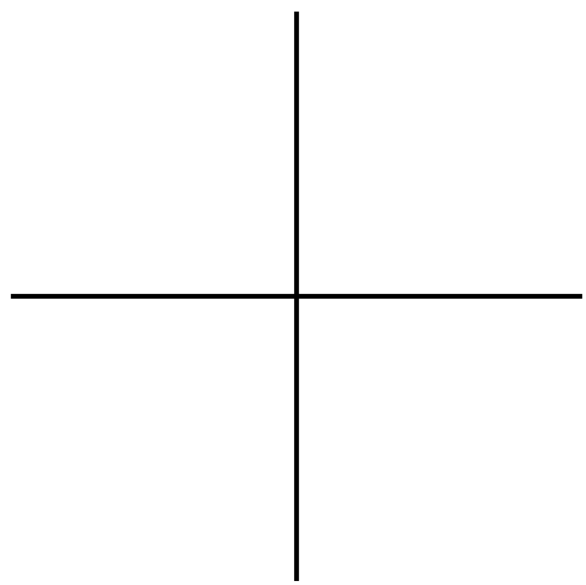
1. Suppose we are attempting to sketch the graph of $f(x) = (x-3)(x-1)(x+2)$.

a. One way we could approach this problem would be to start with a table of values. Complete the table below. Then use the points from your table to make a rough sketch of the graph.

Table

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

Sketch of Graph



b. What are the x-intercepts of this graph? Could we have determined the x-intercepts just by looking at the equation?

c. What important point on the graph is given by $f(0)$?

d. Our table of values only went up to 6. What would happen if we found the values of $f(7)$, $f(8)$, $f(9)$, $f(10)$, etc? Would these values get bigger? Smaller? Can we tell without actually finding the values?

e. In part **d**, we can tell that the value of $f(x)$ is going to increase because as x gets bigger, $x-3$, $x-1$, and $x+2$ all get bigger as well. Look back at your graph. You should see that the right side of the graph goes up.

Will the left side of the graph go up as well? Will it go down? How can we tell?

f. In part **e**, we should have determined that the left side of the graph is going down. Notice that $x-3$, $x-1$, and $x+2$ are all negative numbers on the left side of the graph. Why does this suggest that the left side of the graph will go down? ***Hint:*** What happens if we multiply three negative numbers together?

g. We say that the *end behavior* for the polynomial $f(x)=(x-3)(x-1)(x+2)$ is up on the right and down on the left. Do all polynomials go up on the right and down on the left? Could we have a polynomial that goes up on both sides? Down on both sides? Down on the right and up on the left? ***Note:*** We will explore these questions further in the next few problems, but see what you can figure out on your own first.

2. Here's a new polynomial: $f(x) = (x-4)(x-1)(x+1)(x+5)$. This time, we will attempt to sketch the graph for our polynomial ***without*** making a table.

a. Find the coordinates of the y-intercept for this polynomial.

b. You should have found that the y-intercept is $(0,20)$. (To find the y-intercept, we find $f(0) = (0-4)(0-1)(0+1)(0+5) = 20$.) Now find the coordinates of the x-intercepts.

c. You should have found that the x-intercepts were $(4,0)$, $(1,0)$, $(-1,0)$, and $(-5,0)$. Now we will explore the end behavior for this polynomial. On the right hand side, does the graph go up or down?

d. You should have found that the graph goes up on the right side because all of our factors are positive and if we multiply four positive numbers together, we get a positive. Now determine whether the graph will go up or down on the left side.

e. You should have found that the graph goes up on the left side as well because all four of our factors are negative and if we multiply four negative numbers together, we get a positive. Now sketch the graph of the polynomial. Your graph does not need to be drawn to scale, but it should show the x-intercepts, the y-intercept, and the end behavior.

f. It's often a good idea to check our answers. Graph $y_1 = (x-4)(x-1)(x+1)(x+5)$ on your graphing calculator. To see the graph, you will probably need to adjust the viewing window. ***Hint:*** Is it the x-values or the y-values that need to be large?

g. Here's a good viewing window for the last problem:

x-min: -10, x-max: 10, y-min: -300, y-max: 300

Why do the y-values need to be so much larger than the x-values?

h. Compare your graph in part **e** to the graph on the calculator. Was your graph pretty similar to the picture on the calculator? Is there any way that you should have drawn your graph differently?

3. Yet another polynomial: $f(x) = -2(x-5)(x-3)(x+1)$. Again, we will attempt to sketch the graph for our polynomial without making a table.

a. What's different about this polynomial from the previous two that we investigated?

b. Find the coordinates of the y-intercept and the x-intercepts.

c. You should have found that the y-intercept was $(0,-30)$ and the x-intercepts were $(5,0)$, $(3,0)$, and $(-1,0)$. Is the end behavior on the right side of the graph up or down?

d. You should have found that the end behavior on the right side of the graph was down. The factors $x-5$, $x-3$, and $x+1$ are all positive on the right side of the graph, but they are multiplied by a -2 at the front of the polynomial, so the product is negative. Now determine whether the end behavior on the left side of the graph is up or down.

e. Sketch the graph of $f(x) = -2(x-5)(x-3)(x+1)$. Your sketch should include the x-intercepts, the y-intercept, and the end behavior.

f. Graph this polynomial on your calculator. Make sure you use a good viewing window. (It's okay if you have to try a few different viewing windows before you find a good one.) Does the graph from the calculator match the sketch you made in part **e**?

4. Sketch the graph for $f(x) = -(x-5)(x-2)(x+1)(x+2)$. Your sketch should have correct end behavior and include the coordinates for the x-intercepts and y-intercept. Check your sketch by graphing on the calculator. (Make sure you find a good viewing window when graphing on the calculator.)

5. Now we're going to explore the graph of the polynomial $f(x) = (x+2)(x-3)^2$.

a. What's different about this polynomial from the other polynomials that we've investigated earlier in this handout?

b. This polynomial is different from the other ones that we've studied because the $x-3$ is squared. We say that the factor $x-3$ has a *multiplicity* of 2, whereas the factor $x+2$ has a multiplicity of 1. Our goal in this problem is to determine how different multiplicities affect the graph of the polynomial.

Make an attempt at sketching the graph of $f(x) = (x+2)(x-3)^2$. You should be able to determine the x-intercepts, the y-intercept, and the end behavior, but it might be tricky fitting them all together.

c. Graph $y_1 = (x+2)(x-3)^2$ on your calculator. How does the graph on the calculator compare to the graph that you made? What special thing happens at $(3,0)$?

d. You should have noticed that the graph of $y_1 = (x+2)(x-3)^2$ "just touches" the x-axis at the point $(3,0)$, but then it "bounces" off of the axis. Conversely, the graph "goes through" the x-axis at the point $(-2,0)$. Why do the two x-intercepts work differently?

e. Make an attempt at sketching the graph of $f(x) = (x+2)(x-3)^3$. You should be able to determine the x-intercepts, the y-intercept, and the end behavior.

f. Graph $y_1 = (x+2)(x-3)^3$ on your calculator. How does the graph on the calculator compare to the graph that you made? How do x-intercepts with multiplicity 2 compare to x-intercepts with multiplicity 3? ***Hint:*** Think about "bounce off" versus "go through."

g. Make an attempt at sketching the graph of $f(x) = (x+2)(x-3)^4$. You should be able to determine the x-intercepts, the y-intercept, and the end behavior. Check your sketch by graphing on the calculator.

h. Make an attempt at sketching the graph of $f(x) = (x+2)^2(x-3)^2$. You should be able to determine the x-intercepts, the y-intercept, and the end behavior. Check your sketch by graphing on the calculator.

i. Make an attempt at sketching the graph of $f(x) = (x+2)^2(x-3)^3$. You should be able to determine the x-intercepts, the y-intercept, and the end behavior. Check your sketch by graphing on the calculator.

6. In your own words, describe the role that multiplicity plays in graphing polynomials.

7. Consider the polynomial $f(x) = (x-2)^5(x-1)^4(x+3)^8$

a. Does the graph "bounce off" or "go through" the x-axis at $(2,0)$?

b. Does the graph "bounce off" or "go through" the x-axis at $(1,0)$?

c. Does the graph "bounce off" or "go through" the x-axis at $(-3,0)$?

Answers: a. go through b. bounce off c. bounce off

An important conclusion: We "go through" when the multiplicity is odd and "bounce off" when the multiplicity is even.

8. More practice. Sketch the graph of each of the following polynomials without using your calculator. Your sketches should include the coordinates of the x-intercepts, the y-intercept, and the correct general shape of the graph. Check your answers on the calculator.

a. $f(x) = -3(x-1)^2(x-5)^2$

b. $f(x) = 2(x-4)(x-5)^2$

c. $f(x) = x(x-3)(x+2)$

Hint: This could also be written as $f(x) = (x-0)(x-3)(x+2)$

d. $f(x) = -x^2(x+6)$

Hint: Could you rewrite this the same way we rewrote the last one?

e. $f(x) = -2x^4(x-3)(x-4)^2$

f. $f(x) = x^3 + 7x^2 + 12x$ ***Hint:*** Start by factoring.

g. $f(x) = x^4 - 10x^2 + 9$ ***Hint:*** Always start by factoring.

h. $f(x) = 24x + 22x^2 - 2x^3$

i. $f(x) = x^3 + 3x^2 - 4x - 12$

j. $f(x) = (x^2 - 9)(x^2 + 7x + 12)$

9. The polynomials $f(x) = (x^2 - 3)(x^2 - 5)$ and $f(x) = (x - 3)^2(x - 5)^2$ look similar, but they are ***not*** the same. Sketch the graph for each of these. ***Hint:*** Some of the x-intercepts will involve square roots.