Algebra 2/Pre-Calculus

Name_____

Comparing Exponential and Linear Functions (Exponentials, Day 5)

In this problem set, we will explore similarities and differences between exponential and linear functions. We will also compare exponential functions to power functions.

1. After you graduate from college you get offered a job as a junior executive at Acme Materials Company. Your initial hourly pay rate is $15 and they gave you two different options for what your raises will be. The raises will be given each year right before the end of the year.

Option 1: Each year you get a $4 raise in your hourly pay.
Option 2: Each year you get a 13% raise in your hourly pay (from its current level).

a. Before doing any calculations, determine which option is linear and which option is exponential. Explain how you can tell from the wording of the problem.

b. Write a function formula modeling Option 1.

c. Write a function formula modeling Option 2.

d. In which year does the pay rate for Option 2 surpass the pay rate for Option 1? ***Hint:*** Graph both functions on your calculator.

e. Suppose you work at this job for 40 years. (Unlikely!) How much would you earn according to each option? How realistic is this? Explain.

2. Decide whether each of the following is modeled by an exponential function, a linear function, or some other type of function.

a. Laura is driving at 60 miles per hour.

b. A bank promises a 4.3% annual growth rate.

c. While doing homework, Jessie works at a rate of 3 problems every 10 minutes.

d. The number of bacteria in a Petri dish doubles every two hours.

e. While riding on a Ferris wheel, Ben starts at a height of 10 feet. 40 seconds later, he is at a height of 60 ft.

f. Brian's cell phone plan permits up to 300 text messages each month. He sends 25 texts each day.

3. Twenty years ago, the population of the United States was 300 million. Today, the population is 318 million.

a. Predict what the population will be in ten years. ***Note:*** You must decide whether to use an exponential model, a linear model, or some other model.

b. Which model did you use? (Exponential, linear, or something else.) Why? Explain.

4. When Jerry bought his motorcycle, it was worth $17,000. Five years later, it was worth $6000.

a. Write an exponential function giving the value of Jerry's motorcycle after x years.

b. Write a linear function giving the value of Jerry's motorcycle after x years.

c. When will the value of Jerry's motorcycle fall below $3000? Find the answer according to each model.

d. What is the value of the motorcycle after 15 years? Find the answer according to each model.

e. Which model is more realistic? Explain.

5. Here are two tables of values. One is linear and the other is exponential.

x	$f(x)$
0	5
1	7
2	9
3	11
4	13

x	$g(x)$
0	5
1	10
2	20
3	40
4	80

a. Find formulas for $f(x)$ and $g(x)$.

b. Consider this statement: "Exponential functions have an 'add-multiply' property." Explain what is meant by this statement. Explain how this relates to $g(x)$.

c. Consider this statement: "Linear functions have an 'add-add property." Explain what is meant by this statement. Explain how this relates to $f(x)$.

d. Express $g(x+1)$ in terms of $g(x)$.

e. You should have found that $g(x+1) = 2 \cdot g(x)$. Now do the same thing for $f(x)$: Express $f(x+1)$ in terms of $f(x)$.

6. Each of the following tables shows a function that is either linear or exponential.

x	$f(x)$
0	2
1	8
2	32
3	128
4	512

x	$g(x)$
0	5
1	10
2	20
3	40
4	80

x	$m(x)$
0	900
1	540
2	324
3	194.4
4	116.64

x	$n(x)$
25	101
27	104
29	107
31	110
33	113

a. Write a formula for $f(x)$.

b. Write a formula for $g(x)$.

c. Write a formula for $m(x)$.

d. Write a formula for $n(x)$.

7. Consider the following functions: $f(x) = 5^x$ and $g(x) = x^5$.

a. Are these functions equal? Explain.

b. Sketch a graph for each of these functions.

c. **(Optional Challenge Problem)** As x increases, which will be bigger: $f(x)$ or $g(x)$? How can you tell?

d. **(Optional Challenge Problem)** Which grows faster: An exponential function or a power function? Suppose $e(x) = ab^x$ and $p(x) = ax^n$. Which grows faster: $e(x)$ or $p(x)$? Why?

8. Sketch the graph for each of the following functions. Your graphs do not need to be drawn to scale, but should include the coordinates for the y-intercept and any x-intercept(s). You can check your sketches by graphing on your calculator. ***Suggestion:*** Start by thinking about the type of function. Is it linear? Exponential? Polynomial? Trigonometric?

a. $y = 3(x - 2) + 10$

b. $y = 5(.9)^x$

c. $y = 5x^4$

d. $y = -2(3)^x - 5$

e. $y = x^3 - 36x$

f. $y = 4\cos(x) + 3$